CBCS SCHEME

USN

18CS36

# Third Semester B.E. Degree Examination, June/July 2023
## Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

**Note:** ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

### Module-1

1 a. Indicate how many rows are needed for the truth table of the compound proposition :
$(p \vee \neg q) \leftrightarrow \{(\neg r \wedge s) \rightarrow t\}$
Find the truth value of this proposition if p and r are ture and q, s, t are false. **(07 Marks)**

b. Prove that the following argument is valid :

$$\begin{array}{l} \forall x, [p(x) \rightarrow \{q(x) \wedge r(x)\}] \\ \forall x, [p(x) \wedge s(x)] \\ \hline \therefore \forall x, [r(x) \wedge s(x)] \end{array}$$

**(07 Marks)**

c. Prove that for all integers 'k' and 'ℓ', if 'k' and 'ℓ' are both odd, then $k + \ell$ is even and $k\ell$ is odd by direct proof. **(06 Marks)**

OR

2 a. Prove that for any three propositions p, q, r
$[(p \vee q) \wedge (p \vee \neg q) \vee q] \Leftrightarrow p \vee q$
Using truth table. **(07 Marks)**

b. Test the validity of the argument :

$$\begin{array}{l} P \rightarrow (q \rightarrow r) \\ \neg q \rightarrow \neg p \\ P \\ \hline \therefore r \end{array}$$

**(07 Marks)**

c. Write down the following proposition in symbolic form, and find its negation :
"If all triangles are right – angled, then no triangle is equiangular". **(06 Marks)**

### Module-2

3 a. Prove by mathematical induction that, for any positive integer n
$1 + 5 + 9 + - - - + (4n - 3) = n(2n - 1)$. **(07 Marks)**

b. In the word SOCIOLOGICAL
i) How many arrangements are there for all letters in the word?
ii) In how many arrangements all vowels are adjacent?
iii) In how many arrangements A and G are adjacent. **(07 Marks)**

c. In how many ways can 10 identical pencils be distributed among 5 children in the following cases :
i) There are no restrictions
ii) Each child gets at least one pencil
iii) The youngest child gets atleast two pencils. **(06 Marks)**

**OR**

4 a. Prove by mathematical induction that
$4^n < n^2 - 7$ for all integers $n \geq 6$. **(07 Marks)**

b. Find the coefficient :
i) $x^{12}$ in the expansion of $x^3(1-2x)^{10}$
ii) $xyz^2$ in the expansion of $(2x - y - z)^4$. **(07 Marks)**

c. Find the number of arrangements of all the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's? **(06 Marks)**

## Module-3

5 a. Let $f : R \to R$ and $g : R \to R$ be defined by
$f(a) = 2a + 1$, $g(b) = \frac{1}{3}b$, $\forall a \in R$, $\forall b \in R$
verify that $(gof)^{-1} = f^{-1} o\, g^{-1}$. **(07 Marks)**

b. ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that distance between them is less than ½cm. **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 6, 8, 12\}$. On A, define the partial ordering relation R by xRy if and only if "x divides y". Draw the Hasse diagram for R by verifying R is a partial order on A. **(06 Marks)**

**OR**

6 a. For a fixed integer $n > 1$, prove that the relation "congruent modulo n" is an equivalence relation. **(07 Marks)**

b. Consider the set $A = \{1, 2, 3, 4, 5\}$ and the equivalence relation :
$R = \{(1, 1), (2, 2), (2, 3), (3, 2) (3, 3), (4, 4), (4, 5), (5, 4), (5, 5)\}$
Defined on A. Find the partition of A induced by R. **(07 Marks)**

c. Let f and g be functions from R to R defined by
$f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(g \circ f)(x) = 9x^2 - 9x + 3$.
Determine a and b. **(06 Marks)**

## Module-4

7 a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(07 Marks)**

b. Find the rook polynomial for the board shown below :

| | | | | |
|---|---|---|---|---|
| 1 | 2 | | | |
| | 3 | | | |
| | | 4 | 5 | |
| | | | 6 | 7 |

**(07 Marks)**

c. The number of virus affected files in a system is 1000(initially) and this increases 250% every 2 hours. Use recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

**OR**

8 a. An apple, a banana, a mango and an orange are to be distributed to four boys $B_1$, $B_2$, $B_3$, $B_4$. The boys $B_1$ and $B_2$ do not wish to have apple, the boy $B_3$ does not want banana or mango, and $B_4$ refuses orange. In how many ways the distribution can be made so that no boy is displeased. **(07 Marks)**

b. There are right letters to eight different people to be placed in eight different addressed envelope, Find the number of ways of doing this so that at least one letter gets to the right person. **(07 Marks)**

c. If $a_n$ is a solution of the recurrence relation :

$a_{n+1} = Ka_n$ for $n \geq 0$ and $a_3 = 153/49$, $a_5 = \dfrac{1377}{2401}$, what is K? **(06 Marks)**

## Module-5

9 a. Prove that in every graph, the number of vertices of odd degree is even. **(07 Marks)**

b. Examine whether the following graphs are isomorphic or not. (Refer Fig.Q9(b)).

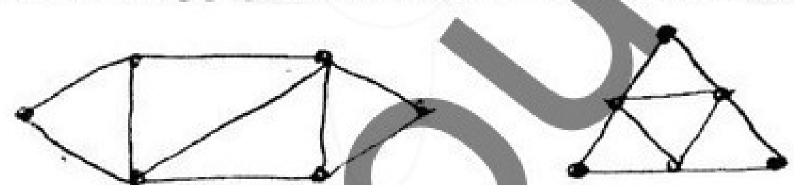

Fig.Q9(b) **(07 Marks)**

c. Apply merge sort to the list.
$-1, 0, 2, -2, 3, 6, -3, 5, 1, 4$. **(06 Marks)**

**OR**

10 a. Construct an optimal prefix code for the symbols a, o, q, u, y, z. **(07 Marks)**

b. Let $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees. If $|E_1| = 19$ and $|V_2| = 3|V_1|$, determine $|V_1|$, $|V_2|$ and $|E_2|$. **(07 Marks)**

c. Show that there is no graph with 28 edges and 12 vertices in the following cases :
i) The degree of a vertex is either 3 or 4
ii) The degree of a vertex is either 3 or 6. **(06 Marks)**

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